On Neuberg

Ian McKinnon, September 2019

Introduction

Bridge players are probably not aware of the way in which their matchpoint scores are calculated for boards that are not played as many times as others in many events. This may occur where a board has been fouled, an average is awarded by the director or a player becomes ill and has to leave the movement for a round or two.

At the club level it is more often due to the movement dictating the situation where some boards are played a different number of times. In these cases the results of all the players in the section are impacted significantly. This is unreasonable and completely unnecessary.

The important and overriding requirement of any bridge tournament is that all boards in the event should be of equal value. Therefore where some results are missing from a board, a method must be adopted to estimate the scores achieved by the players that is fair and equitable. It must be fair to the pairs that played the board and those that did not.

In 1991 the WBF, ACBL and others chose to adopt a method using the Neuberg formula. Myself and other scoring program writers at the time were not aware of, disagreed with or chose to ignore this arbitrary decision. It seems to be clear that the mathematics of this decision was not extensively debated by the administrators and they chose to ignore those that disagreed with their decision. The mathematics of this issue are not simple and generally is not understood by most people.

David desJardins PHD, Mathematics, Berkley University was one of the outspoken critics of this move to the *Neuberg Method*. When asked by one of his correspondents why he had not taken this further with the WBF scoring committee he said '*I decided that if I can't even convince you, the chances of successfully explaining the issues to the WBF scoring committee were close to zero, so I gave up.*'

So there we have it. A method that has been in use for about 27 years that not everyone supports. Therefore I would like to take a look at the impacts this estimating method has on bridge results. Perhaps I can show that we are not always being treated well by this particular method and that one recommended alternative is simpler, statistically correct and very easy to understand. This alternative is called the *Factor Method*.

As well as the *Factor Method* there are a number of other possibilities. All these methods of estimating the matchpoints on incomplete boards has its proponents, each voicing their reasons very strongly.

See more details on this subject in ***Further reading***. If you are not interested in the following mathematics just skip your reading to the ***First example***.

The Neuberg Method announcement

The WBF formula for fouled boards and adjusted scores was reported in the ACBL Bulletin June 1991 pages 15 and 16:

The basic formula is: M = (N x S)/n + (N - n)/2n
Where:
M=final matchpoints on board
N=number of scores on the board
S=matchpoint score in group
n=number of scores in group
This applies to groups of 4 or more scores
It also applies to a group of 3 if it is the largest group (else 70%,60%,50%)
Group of 2 gets 65% and 55%
Equal scores share arbitrary scores
Single scores get 60% rounded to nearest 10th.
Where averages are awarded, the other pairs in the field have this formula applied.

This is the *Neuberg Method* and it is applied to any board which has two or more sets of unrelated scores. Note also the formula above is shown for single matchpointing.

The generic estimation formula

I approached my friend Peter Buchen (mathematician and Honorary Associate Professor, Sydney University) earlier this year to help me convince the administrators of bridge that the existing estimation of matchpoints for incomplete boards was questionable.

His findings were very surprising to me at the very least. Please see Peter's complete paper in ***Further reading***.

Using the same variables and nomenclature as above for the Neuberg formula, Peter shows that the generic formula for the estimation of the matchpoints is:

$$M = (N - k) / (n - 1) \times S + \tfrac{1}{2} (k - 1)$$

The only new variable introduced here is k. By selecting different values of k we arrive at all the existing matchpoint estimation formulae and we can choose to think up as many new ones as we like. The values of k for most of the existing published methods are as follows:

Neuberg method:	$k = N / n$
Ascherman method:	$k = (N - 1) / (n + 1)$
Factor method:	$k = 1$
Manual method:	$k = N - n + 1$

Let me introduce you to another option. I chose this because I thought it has some very useful properties. All existing methods were chosen by the inventors for similar reasons. Essentially there are just as many sound mathematical reasons for mine as any other. I believe this method is at least as good as Neuberg, if not better.

McKinnon method
k = (N – 1) / n where N > n+1 otherwise k = 1

These values of k when applied to the generic estimation formula, and simplified, leads to the following formulae (using single matchpointing) for each method:

Neuberg method:	M = N x (S + ½) / n – ½
Ascherman method:	M = (N – 1) x (S+ ½) / n
Factor method:	M = (N – 1) x S / (n – 1)
Manual method:	M = S + (N – n) / 2
McKinnon method:	M = (N / n + 1 / n / (n-1)) x (S + ½) – n /2(n – 1)

The differences

Statistically speaking these methods are essentially the same as each other. The one thing that does differ is the ***variance.*** A lower variance means the scores are less spread out. Generally you will find a 50% score in all methods remains at 50% because of the symmetry about 50%. The maximum difference between the methods will occur at precisely the top and bottom scores. Since tops are very important to the ultimate winner, this difference could be very significant. Whichever method is used the check-total of all the scores on the board is constant and equal to any unaffected board.

In the *Neuberg method* above-average scores are reduced, and below-average scores are increased. The amount reduced and increased varies with the distance from 50%. This particularly effects the results for groups of scores where the number of scores are low.

In the *Ascherman method* the scores are distributed evenly and have the same percentages irrespective of the number of scores on the board (N). The top and bottom scores are also evenly positioned away from the 100% and 0% marks.

In the *Factor method* the scores are distributed evenly and have the same percentages as those achieved in the original playing of the board. A top score in the original playing delivers a top on the board.

In the *Manual method* the scores are distributed evenly within their group of n scores and the top scores are punished severely while the bottom scores are rewarded unduly. Small groups of scores are severely affected. As the name suggests this method was used prior to the use of computers for scoring.

The *McKinnon method* has the same advantages and disadvantages as the Neuberg method but with the additional advantage of delivering final scores on the board closer to the top and bottom as those achieved during the original playing of the board. That is, the variance is greater than that of Neuberg. Also, most significantly, when N = n + 1 the results are the same as the Factor method.

First example

Consider a board where a pair has received an average score awarded by the Tournament Director. Say there are normally 9 results on each board so that once the average has been awarded there are only 8 scores left. This table shows the original matchpoint scores (MPs) and percentages. In addition it shows the various popular estimation methods once the board is complete, based on 9 results. The estimation methods shown are Factor (F), McKinnon (Mk), Neuberg (N), Ascherman (A) and Manual (M). The corresponding percentages are also shown. The matchpoint scores are shown using double matchpointing but halving those delivers the single matchpoint scores.

Score	MPs	%	F	F%	Mk	Mk%	N	N%	A	A%	M	M%
420	14	100	16	100.0	16	100.0	15.8	99.25	15	93.75	15	93.75
400	12	85.7	13.71	85.69	13.71	85.69	13.6	85.13	13	81.25	13	81.25
300	10	71.4	11.43	71.44	11.43	71.44	11.3	71.13	11	68.75	11	68.75
200	8	57.1	9.14	57.13	9.14	57.13	9.12	57.00	9	56.25	9	56.25
100	6	42.9	6.86	42.88	6.86	42.88	6.88	43.00	7	43.75	7	43.75
-50	4	28.6	4.57	28.56	4.57	28.56	4.62	28.88	5	31.25	5	31.25
-100	2	14.3	2.29	14.31	2.29	14.31	2.38	14.88	3	18.75	3	18.75
-150	0	0	0	0.00	0	0.00	0.12	0.75	1	6.25	1	6.25

Observations:

- The Factor and McKinnon methods deliver the same percentages as the original playing.
- The Neuberg method delivers a slightly lower top and higher bottom.
- The Ascherman and Manual methods deliver significantly lower top and higher bottom.
- The last three methods have a lower variance (less spread of the scores) which means the better scores are punished and the lower scores rewarded for no particular reason.

As bridge players we are very happy to play in an 8 table movement and win or lose as the case may be and accept the scores produced by the standard matchpointing. That begs the question: why should the awarding of an average score at one table effect the other results so significantly using anything other than the Factor and McKinnon Methods? All the other pairs had no influence over the awarded average result just like the average score should not have any influence over the other scores.

Second example

Consider a board where two pairs have received an average score (or similar) awarded by the Tournament Director. This could be because the board was fouled for two rounds and the director felt that two results cannot be used for estimation of the final scores on the board. So there are normally 9 results on the board so that once the average has been awarded there are only 7 scores left.

Score	MPs	%	F	F%	Mk	Mk%	N	N%	A	A%	M	M%
400	12	100	16	100.0	15.86	99.13	15.7	98.19	14.86	92.88	14	87.50
300	10	83.3	13.33	83.31	13.24	82.75	13.1	82.13	12.57	78.56	12	75.00
200	8	66.6	10.67	66.69	10.62	0.44	10.5	66.06	10.29	64.31	10	62.50
100	6	50.0	8	50.00	8	50.00	8	50.00	8	50.00	8	50.00
-50	4	33.3	5.33	33.31	5.38	33.63	5.43	33.94	5.71	35.69	6	37.50
-100	2	16.6	2.67	16.69	2.76	17.25	2.86	17.88	3.43	21.44	4	25.00
-150	0	0	0	0.00	0.14	0.88	0.29	1.81	1.14	7.13	2	12.50

The impact of all the estimations are much the same as in example 1 but with the following observations.

Observations:

- The Factor method deliver the same percentages as the original playing.
- From left to right each other method has less variance
- The McKinnon method delivers a slightly lower top and higher bottom with 99.13% for the top.
- The Neuberg method similarly delivers lower top and higher bottom but the top is 98.19%.
- The Ascherman methods delivers a 92.88% top.
- The Manual methods delivers an 87.50% top.
- All but the Factor method reduce the variance so that all the scores move closer and closer towards the average from left to right.

In choosing the Neuberg method the bridge administrators have decided, for no obvious reason, that it delivers the best variance.

Third example

Consider an example of applying the *Neuberg Method* to a fouled board. Let us assume this example applies to a board from an event with 13 tables and the board was fouled after 5 rounds (the cards were returned to the wrong slot making the board essentially a different hand).

Here the fouled board has 5 results in one group (1) and 8 in the other (2). In the table below the *Neuberg Method* matchpoints are shown as well as the original matchpoint scores (MPs). The *Neuberg Method* percentages are based on a maximum score of 24 matchpoints and the others on the original maxima of 8 and 14. The *Factor Method* (F) is shown in the last column and has the same percentage as the original independent scores.

Scores(1)	Neuberg(1)	Neuberg %(1)	MPs(1)	MPs%(1)	F(1)
360	22.4	93.33	8	100	24
140	17.2	71.67	6	75	18
130	12	50	4	50	12
110	6.8	28.33	2	25	6
100	1.6	6.67	0	0	0
Scores(2)	Neuberg(2)	Neuberg %(2)	MPs(2)	MPs%(2)	F(2)
50	21.8	90.83	13	92.83	22.29
50	21.8	90.83	13	92.83	22.29
-110	16.9	70.42	10	71.42	17.14
-140	7.1	29.58	4	28.57	6.86
-140	7.1	29.58	4	28.57	6.86
-140	7.1	29.58	4	28.57	6.86
-140	7.1	29.58	4	28.57	6.86
-140	7.1	29.58	4	28.57	6.86

These two groups of 5 and 8 scores illustrate how poorly the smaller groups do with the *Neuberg Method*. Considering the percentage scores, the first place in the group of 5 suffers by 667 basis points and the last place gains by 67 basis points. Likewise in the group of 8 the tied first place suffers a loss of 200 and last tied places gains by 101. This illustrates how difficult it is for a pair to score well in a small group when the *Neuberg Method* is used.

F(1)&F(2)	MPs	%
24	24	100
22.29	21	87.5
22.29	21	87.5
18	18	75
17.14	16	66.7
12	14	58.33
6.86	8	33.33
6.86	8	33.33
6.86	8	33.33
6.86	8	33.33
6.86	8	33.33
6	2	8.33
0	0	0

Let us consider what the Factor method does deliver. You receive the same percentage on the board as that achieve by you when played in the group of lesser pairs. As bridge players we are very happy to play in a 5 table movement and we win or lose the event and accept the scores produced by matchpointing. Essentially we are happy with the sample of 5 scores and the top, bottom or otherwise we score is based on that sample.

When we play a fouled board our play and expectations are the same as when we play any other board. At the time of playing the board we are not aware of the fouled nature of the board. We should expect that after an estimation has been applied to the board because of the fouling, that we receive the same percentage on the board as we achieved in the playing in the group of 5 as in this example. It defies logic to receive anything more or less than the original score. On top of this to receive a score based on an arbitrary formula chosen by an administrator is even more bizarre.

With the Factor method you will also see that with two groups there is a likelihood that there will be two pairs receiving a top on the one board. At first this may appear to be strange but I am sure the two pairs with a top (or bottom or any other score) will feel they deserve that score. Again the fact that the board is fouled should not influence the outcomes of the results. Pairs so effected and who played the board with good intent and expectations should be rewarded for their efforts.

Fourth example

Let us consider another scenario that is not that uncommon. In fact at the club level where most of our bridge is played, this example illustrates the most likely case where estimation of scores will happen. Above in the ACBL published article it states: '*Where averages are awarded, the other pairs in the field have this formula applied.*' Therefore if a board is played one time less than other boards in the event then the *Neuberg Method* is applied to the existing scores.

Say a club usually plays about 26 boards in a session of bridge and they normally run a two winner event. In this example we have 13 and 1/2 tables (27 pairs). From his point-of-view the director considers the best option is to run this as a 14 Table Mitchell with a skip at half way and the EW pair missing at one table though it could be a NS pair. There are 13 rounds. This movement ticks all the boxes: easy to run, no shared boards, it completes after playing 26 boards and pairs only sit out for 2 boards.

With 2 boards on each table there are 28 boards in play so clearly no one plays all boards. Some pairs will play 26 boards and some, those that sit out for one round, play only 24 boards. There are 13 pairs that sit out for one round. The other 14 pairs play every round, those pairs being one NS pair and 13 EW pairs. Of the 14 sets of two boards, one set is played 13 times and the rest 12 times.

Using double matchpoint scoring the top on one set is 24 (12 single MP) and the other 13 sets the top is 22 (11). In other words, 26 boards have a top of 22 (11) and two boards a top of 24 (12).

It is paramount to make all boards of equal value. Therefore we need to recalculate the matchpoints on those 26 boards as if the pairs *played them 13 times*.

The pairs that played those 26 boards and scored a top, did in fact *score a top*. There is no doubt they did score a top. There is no way in this event that the pairs that did score this top can ever have that top beaten because the event finished after 13 rounds.

So if they got a top they received 22 (11) MPs because the board was played 12 times. To match the two boards that were played 13 times their new top should be 24 (12). If you rescore these boards using the *Neuberg Method* then they score 23.9, not 24. This cannot be right or fair.

I suggest the normal for this event is not a top of 24 (12) but it is really 22 (11). There are 26 boards with a top of 22 and two boards with a top of 24. The two boards that were played 13 times were the exception in this event rather than the rule.

Similarly the pairs that scored a top on the two boards with a top of 24 (12) did, in no doubt, score a top. In the same way as the other 26 boards when you attempt to establish equity you cannot take that top away from them.

Using the variables listed above in the *Factor Method* definition, consider this example of one board where N=13 and n=12 (thus considering a board of 12 scores that are to be estimated equivalent to a board with 13 scores). Note the percentages are the same before and after applying the *Factor Method*. That is not the case with the *Neuberg Method* shown in the right hand two columns.

Score	MPs	%	F	F%		Neuberg	%(of 24)
420	22	100	24	100		23.92	99.67
400	20	90.9	21.81	90.9		21.75	90.6
300	18	81.8	19.64	81.8		19.58	81.6
250	16	72.7	17.45	72.7		17.42	72.6
200	14	63.6	15.27	63.6		15.25	63.5
100	12	54.5	13.09	54.5		13.08	54.5
80	10	45.5	10.91	45.5		10.92	45.5
50	8	36.4	8.73	36.4		8.75	36.5
-50	6	27.3	6.55	27.3		6.58	27.4
-100	4	18.2	4.36	18.2		4.42	18.4
-120	2	9.1	2.18	9.1		2.25	9.4
-150	0	0	0	0		.08	0.3

To illustrate a different approach to this problem, why not consider reducing the scores on the boards with 13 results to those with 12 results? So long as the matchpoint scores are still equal (as percentages) before and after there is nothing lost. Therefore now we use N=12 and n=13 (that is, consider starting with a board of 13 scores and factor <u>down</u> to those with 12 scores).

Score	MPs	%	F
450	24	100	22
420	22	91.67	20.17
400	20	83.33	18.33
300	18	75	16.5
250	16	66.67	14.67
200	14	58.33	12.83
100	12	50	11
80	10	41.67	9.17
50	8	33.33	7.33
-50	6	25	5.5
-100	4	16.67	3.67
-120	2	8.33	1.83
-150	0	0	0

If we were to apply the *Neuberg Method* to this scenario we get the following. For obvious reasons it is not sensible to do this but it does illustrate how strange the method is under these circumstances. The variance is now greater than the original.

Score	M_n	%	Neuberg
450	24	100.004	22.08
420	22	91.96	20.23
400	20	83.55	18.38
300	18	75.18	16.54
250	16	66.77	14.69
200	14	58.41	12.85
100	12	50	11
80	10	41.59	9.15
50	8	33.23	7.31
-50	6	24.81	5.46
-100	4	16.41	3.61
-120	2	8	1.76
-150	0	-.004	-0.08

Some observations.

When using the *Factor Method* it does not matter whether we factor the boards with 12 scores up to 13 scores or the boards with 13 scores down to those with 12 scores. The estimated scores have the same percentages as the original.

By factoring <u>down</u> the two boards with 13 scores we retain the original scores of the majority of the other boards (26 of the 28 in the example above). Given that the *Factor Method* factoring <u>down</u> does retain the majority of the original scores and at the same time produces the same percentages as the *Factor Method* when factoring up, then how can it be wrong to use the *Factor Method* when we need to estimate the scores?

It should be noted that the *Neuberg Method* matchpoint scores are **lower** than *Factor Method* matchpoint scores when the scores achieved by the players are above average and **higher** when below average. David desJardins says '*Neuberg's formula means that above-average scores are reduced if factored up, and below-average scores increased;* ***this makes it harder to win in a smaller section***.'

When estimating the scores of the 26 adjusted boards using *Neuberg Method* the total matchpoint scores of all the pairs are effected significantly. Those pairs doing well could be penalized by 15-20 basis points. The pairs doing badly could be rewarded by 15-20 basis points. In any single session this may not be that significant because the order of the places will not change but in multi-session events this could easily make a difference at the podium.

Groups with low numbers of scores

In example 3 above there were two groups of pairs on the one boards, one group with 8 scores and the other with 5. There it was argued that the smaller group deserved just as much consideration as the larger one. Also it was noted that the smaller group was significantly punished when their score was estimated using the Neuberg method.

In the WBF decree of 1991 it states: [Neuberg] also applies to a group of 3 if it is the largest group (else 70%, 60%, 50%). A group of 2 gets 65% and 55%.

Here is an example of how 3 results look in an event where there are 5 tables. The 3 table group is the larger and accordingly Neuberg applies.

Score	MPs	%	F	F%	Mk	Mk%	N	N%	A	A%	M	M%
-50	4	100	8	100	7.67	95.88	7.33	91.63	6.67	83.38	6.00	75.00
-100	2	50	4	50	4	50.00	4	50.00	4	50.00	4.00	50.00
-150	0	0	0	0	0.33	4.13	0.67	8.38	1.33	16.63	2.00	25.00

It is hard to imagine why Neuberg is any better than any other choice, particularly the Factor method. The same arguments supporting the Factor method still apply here. In fact if the group of 3 is the smallest group there is no reason that I can see for the Factor method still not applying. The arbitrary 70-60-50% seems to be quite strange. The 3 score sample is still ample enough to produce a result. In a board-a-match event 2 results are fine, so why not in an event with 3 results?

Conclusions

The *Neuberg Method* is not necessarily the best candidate for calculating the matchpoint awards where there are different numbers of scores recorded on a variety of boards. The *Factor Method* is simple, accurate and easily understood by the players. The same cannot be said about the *Neuberg Method*.

When using the Factor Method the scores are correct and all boards are of equal value. This is not always the case when using the *Neuberg Method.*

One thing that is overlooked by the administrators is the rarity for the need to use matchpoint estimations. The simplest common need is the awarding of average or adjusted scores on the occasional board. This rarely effects more than one result on a board. When using the Neuberg method all the other results on the board are affected significantly. This is clearly unfair to all pairs except, perhaps, the pair receiving the adjustment.

The most common occurrence (in terms of the number of scores affected) is that illustrated by example 4 above. It is the case where the movement dictates the unequal number of scores on some boards in the movement. Again, this effects all the pairs in the movement without any clear reasons for using Neuberg.

The *Factor Method* can also be used where the number of results on all boards are the same but where the results need to be presented in a way that is familiar to the players. For example if the players are familiar with seeing the scores with a top of 24(12) then the scores for ***any*** movement can be factored to 13 tables without any impact on the final percentage scores.

Another advantage of factoring the scores to a predetermined number of tables is the results in multi-session events can be tallied from session to session using the matchpoint scores even where there are different numbers of tables in each session. This is an understandable way of presenting the scores to the players and lends itself to simple carried-forward matchpoint scores from one stage to the next. When scores are presented as percentages this does not have the same flexibility and comprehension by the players.

The most common example of where there is a need to estimate scores is where the movement dictates the need. Example 4 above illustrates this case. Clearly the Neuberg method in this case is unfair and the Factor method should be used.

The alternative of using the McKinnon method addresses the example 4 problem (producing the same results as the Factor method) as well as producing a greater variance in all other cases.

So why is the *Neuberg Method* being used? In March 1983, when the ACBL and I were developing our first scoring programs for PCs, we had discussions at the Spring Nationals on this very issue. We decided that the *Factor method* was clearly the best way to go with so many advantages as outlined above. Why has this changed?

It is doubtful that the WBF scoring committee invited any opinions from any statisticians or mathematicians other than Neuberg in 1991. Ultimately the choice of method is arbitrary and based on a variety of opinions. That said, I believe there are very good reasons for using more than one method, each applying to different situations.

Also I have shown there is a new alternative method that is essentially somewhere between the Neuberg and Factor methods. It supplies the same percentage as the original when there is only one score missing and a greater variance than the Neuberg method in all other cases. The case of the one score missing is by far the most common example of the problem.

I believe it is time for the administrators to reconsider this problem.

Further reading

View this link to see Peter Buchen's definitive paper on the mathematics of this subject.
http://www.asecomputing.com/Downloads/Neuberg.pdf

To consider the details and the mathematics of matchpoint scoring see a comprehensive coverage of the subject by Peter Smulders at:
http://www.pjms.nl/NEUBERG/index_en.htm.

Within that discussion the Ascherman method is highly praised. See the details here:
http://www.pjms.nl/NEUBERG/Ascherman_en.html

To further consider the application of the Neuberg Formula see the explanation given by Max Bavin at:
https://www.ebu.co.uk/documents/laws-and-ethics/articles/neuberg-formula.pdf

Note that Max Bavin says: *'All World Bridge Federation, European Bridge League and English Bridge Union events are scored using the Neuberg formula..'*. This is clearly not all the world and the list certainly does not include a number countries where a lot of bridge is played.

In addition Max Bavin claims '*All – or nearly all – modern computer scoring software has the Neuberg formula already built-in*'. I have found this not to be true though it is found in ACBLScore .

The WBF, EBL and EBU generally conduct events where the number of entries are controlled and highly predictable from an organization point of view. The need to apply estimations of scores is far less likely than found in local club games. That is a very large number of bridge games.

One older alternative called ‘The Manual Method’ is mentioned by Peter Smulders in: http://www.pjms.nl/NEUBERG/manualmethod.html. Therein it is explained that this method is flawed and should not be used.

For an early discussion of the *Factor Method* see the text by David desJardins at: http://www.desjardins.org/david/factor.txt